

Prediction of Leptonic CP Phase in A_4 symmetric model

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Abstract

We consider minimal modifications to tribimaximal (TBM) mixing matrix which accommodate non-zero mixing angle θ_{13} and CP violation. We derive four possible forms for the minimal modifications to TBM mixing in a model with A_4 flavor symmetry by incorporating symmetry breaking terms appropriately. We show how possible values of the Dirac-type CP phase δ_D can be predicted with regards to two neutrino mixing angles in the standard parametrization of the neutrino mixing matrix. Carrying out numerical analysis based on the recent updated experimental results for neutrino mixing angles, we predict the values of the CP phase for all possible cases. We also confront our predictions of the CP phase with the updated fit.

I. INTRODUCTION

Establishing leptonic CP violation (LCPV) is one of the most challenging tasks in future neutrino experiments [1]. The relatively large value of the reactor mixing angle measured with a high precision in neutrino experiments [2] has opened up a wide range of possibilities to explore CP violation in the lepton sector. The LCPV can be induced by the PMNS neutrino mixing matrix [3] which contains, in addition to the three angles, a Dirac type CP violating phase in general as it exists in the quark sector, and two extra phases if neutrinos are Majorana particles. Although we do not yet have compelling evidence for LCPV, the current global fit to available neutrino data indicates nontrivial values of the Dirac-type CP phase [4, 5]. In this situation, it must deserve to predict possible size of LCPV detectable through neutrino oscillations. From the point of view of *calculability*, much attention has been paid to the prediction of the Dirac type LCPV phase with regards to some observables [6]. Recently, it has been shown [7] that Dirac-type leptonic CP phase can be particularly predictable in terms of neutrino mixing angles in the standard parameterization of PMNS mixing matrix [8].

Before the measurements of the reactor mixing angle, the fit to neutrino data was consistent with the so-called tribimaximal (TBM) neutrino mixing matrix, U_0^{TBM} , which is theoretically well motivated flavor mixing pattern [9]. However, it should be modified to accommodate non-zero reactor mixing angle as well as CP violation. Although the current neutrino data rule out the exact TBM mixing pattern, it can be regarded as leading order approximation. Among various possible modification to U_{TBM} , as discussed in [7], the minimal modification is useful to predict Dirac type CP phase. The minimal modification is to multiply U_0^{TBM} by a rotation matrix in the (i, j) plane with an angle θ and a CP phase ξ , $U_{ij}(\theta, \xi)$, whose form is given either $U_{ij}^\dagger(\theta, \xi)U_0^{\text{TBM}}$ or $U_0^{\text{TBM}}U_{ij}(\theta, \xi)$ [10]. Among them, $U_{23}^\dagger(\theta, \xi)U_0^{\text{TBM}}$ and $U_0^{\text{TBM}}U_{12}(\theta, \xi)$ are ruled out because they lead to zero reactor mixing angle. So, all possible forms of minimal modification to TBM mixing matrix are as follows:

$$V = \begin{cases} U_0^{\text{TBM}}U_{23}(\theta, \xi) & \text{(Case-A),} \\ U_0^{\text{TBM}}U_{13}(\theta, \xi) & \text{(Case-B),} \\ U_{12}^\dagger(\theta, \xi)U_0^{\text{TBM}} & \text{(Case-C),} \\ U_{13}^\dagger(\theta, \xi)U_0^{\text{TBM}} & \text{(Case-D).} \end{cases} \quad (1)$$

While the study in [7] has not accounted for the origin of such modification to U_0^{TBM} ,

in this paper, we first study how such a minimally modified TBM mixing pattern can be achieved in a neutrino model with A_4 flavor symmetry by incorporating A_4 symmetry breaking terms appropriately. Then, following [7], we investigate how the Dirac type CP phase can be predicted based on the updated fit results for neutrino mixing angles [5]. As shown later, comparing with the results obtained in [7], the Dirac type CP phase predicted based on the updated fit results has different implication particularly at 1σ C.L.

II. MINIMAL MODIFICATIONS TO TRI-BIMAXIMAL MIXING IN A_4 SYMMETRIC MODEL

In [11], an A_4 symmetric model for neutrino masses and mixing has been proposed to accommodate non-zero mixing angle θ_{13} on top of TBM mixing. Based on the A_4 symmetric model, we study how the forms given in Eq.(1) can be derived by incorporating appropriate A_4 symmetry breaking terms.

A. Case-A

As proposed in [11], A_4 flavor symmetry allows the charged-lepton mass matrix to be diagonalized by the Cabibbo-Wolfenstein matrix [12],

$$U_{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (2)$$

where $\omega = e^{2\pi i/3}$, with three independent eigenvalues, m_e, m_μ, m_τ . This can be realized by the lepton assignments, $L_i = (\nu_i, l_i) \sim \underline{3}$, $l_1^c \sim \underline{1}$, $l_2^c \sim \underline{1}'$, $l_3^c \sim \underline{1}''$ with 3 Higgs doublets $\Phi_i = (\phi_i^0, \phi_i^-) \sim \underline{3}$. Introducing 6 heavy A_4 Higgs singlets and triplet:

$$\eta_1 \sim \underline{1}, \quad \eta_2 \sim \underline{1}', \quad \eta_3 \sim \underline{1}'', \quad \eta_{i(=4,5,6)} \sim \underline{3}, \quad (3)$$

where $\eta_i = (\eta_i^{++}, \eta_i^+, \eta_i^0)$, one can obtain the neutrino mass matrix in the A_4 basis [11]

$$M_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix}, \quad (4)$$

where a comes from $\langle \eta_1^0 \rangle$, b from $\langle \eta_2^0 \rangle$, c from $\langle \eta_3^0 \rangle$, d from $\langle \eta_4^0 \rangle$, e from $\langle \eta_5^0 \rangle$, f from $\langle \eta_6^0 \rangle$. To achieve TBM mixing pattern of the neutrino mixing matrix, A_4 flavor symmetry should be broken to Z_2 in such a way that $b = c$ and $e = f = 0$. Then, the neutrino mass matrix in the flavor basis where the charged lepton mass matrix is diagonal is given by

$$M_\nu^{(e,\mu,\tau)} = U_{CW}^\dagger M_\nu U_{CW}^* = \begin{pmatrix} a + (2d/3) & b - (d/3) & b - (d/3) \\ b - (d/3) & b + (2d/3) & a - (d/3) \\ b - (d/3) & a - (d/3) & b + (2d/3) \end{pmatrix}, \quad (5)$$

which is diagonalized by the TBM mixing matrix U_0^{TBM} . To achieve non-zero mixing angle θ_{13} so as to accommodate neutrino data from reactor experiments, we take $b = c$ and $e = -f \equiv \epsilon \neq 0$ in Eq.(4), and then the neutrino mass matrix in the A_4 basis is given by

$$M_\nu = \begin{pmatrix} a + 2b & \epsilon & -\epsilon \\ \epsilon & a - b & d \\ -\epsilon & d & a - b \end{pmatrix}. \quad (6)$$

In the flavor basis, the neutrino mass matrix can be rewritten as

$$M_\nu^{(e,\mu,\tau)} = \begin{pmatrix} a + (2d/3) & b - (d/3) & b - (d/3) \\ b - (d/3) & b + (2d/3) & a - (d/3) \\ b - (d/3) & a - (d/3) & b + (2d/3) \end{pmatrix} + \frac{i}{\sqrt{3}} \begin{pmatrix} 0 & -\epsilon & \epsilon \\ -\epsilon & -2\epsilon & 0 \\ \epsilon & 0 & 2\epsilon \end{pmatrix}. \quad (7)$$

Rotating the mass matrix given in Eq.(7) by TBM mixing matrix, we get

$$\begin{pmatrix} a - b + d & 0 & 0 \\ 0 & a + 2b & X \\ 0 & X & b - a + d \end{pmatrix}, \quad (8)$$

where $X = \sqrt{2}i\epsilon$ and non-zero entries are complex in general. It can be easily shown that the mass matrix given by Eq.(7) can be diagonalized by

$$V' = U_0^{\text{TBM}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta e^{-i\xi} \\ 0 & \sin \theta e^{i\xi} & \cos \theta \end{pmatrix} \cdot P_\beta, \quad (9)$$

where $P_\beta = \text{Diag}[e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}]$.

Now, let us check testability of the cases in this neutrino model by taking into account the sum-rules among the light neutrino masses [13]. In the leading order, the mass eigenvalues are given by $m_1^0 = a - b + d$, $m_2^0 = a + 2b$, $m_3^0 = b - a + d$, and thus we get the mass sum rules

$$\begin{aligned} m_3^0 &= m_2^0 + m_1^0, \quad \text{for } a = 0, \\ m_1^0 &= 2m_2^0 + m_3^0, \quad \text{for } b = 0. \end{aligned} \quad (10)$$

Including the perturbation given by the second matrix in Eq.(7), we get the following sum rule,

$$\hat{m}_2 + \hat{m}_3 = \hat{m}_2^0 + \hat{m}_3^0, \quad (11)$$

where $\hat{m}_2 \equiv m_2 e^{-i\xi}$, $\hat{m}_3 \equiv m_3 e^{i\xi}$, $\hat{m}_2^0 \equiv m_2^0 e^{-i\xi}$, $\hat{m}_3^0 \equiv m_3^0 e^{i\xi}$ with $m_{i(=1,2,3)}$ representing the mass eigenvalues obtained by diagonalizing the mass matrix Eq.(8). Plugging Eq.(10) into Eq.(11), we can get the following sum rules for $\xi = 0, \pi, 2\pi$,

$$\begin{aligned} m_1 + m_2 - m_3 &= 2 \delta m_2, \quad \text{for } a = 0, \\ 2m_2 + m_3 - m_1 &= \delta m_2, \quad \text{for } b = 0, \end{aligned} \quad (12)$$

where $\delta m_2 \equiv m_2 - m_2^0$ and we have used $m_1 = m_1^0$. The sum rules for $\xi = \pi/2, (3\pi/2)$ are

$$\begin{aligned} m_1 + m_2 &= m_3, \quad \text{for } a = 0, \\ 2m_2 + m_3 - m_1 &= 3 \delta m_2, \quad \text{for } b = 0. \end{aligned} \quad (13)$$

B. Case-B

To realize the case B, we add the breaking terms δM_ν to M_ν in the A_4 basis, which is given by

$$\delta M_\nu = \begin{pmatrix} g+h & 0 & 0 \\ 0 & \omega g + \omega^2 h & 0 \\ 0 & 0 & \omega^2 g + \omega h \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & -A \end{pmatrix}, \quad (14)$$

where $g = -h$ and $A = \sqrt{3}ig$.

Then, the matrix given in Eq.(14) becomes in the flavor basis as follows:

$$\begin{pmatrix} 0 & g & -g \\ g & -g & 0 \\ -g & 0 & g \end{pmatrix}. \quad (15)$$

Then, the mass matrix $M_\nu + \delta M_\nu$ can be diagonalized by

$$V = U_0^{\text{TBM}} \begin{pmatrix} \cos \theta & 0 & -\sin \theta e^{-i\xi} \\ 0 & 1 & 0 \\ \sin \theta e^{i\xi} & 0 & \cos \theta \end{pmatrix} \cdot P_\beta. \quad (16)$$

For the case B, the sum rules at the leading order are the same as Eq.(10). Including the perturbation given by the second matrix in Eq.(15), we get the following sum rule

$$\hat{m}_1 + \hat{m}_3 = \hat{m}_1^0 + \hat{m}_3^0, \quad (17)$$

where $\hat{m}_1 \equiv m_1 e^{-i\xi}$, $\hat{m}_3 \equiv m_3 e^{i\xi}$, $\hat{m}_1^0 \equiv m_1^0 e^{-i\xi}$, $\hat{m}_3^0 \equiv m_3^0 e^{i\xi}$, with $m_{i(=1,2,3)}$ representing the mass eigenvalues obtained by diagonalizing the mass matrix $M_\nu + \delta M_\nu$. Plugging Eq.(10) into Eq.(17), we can get the following sum rules for $\xi = 0, \pi, 2\pi$,

$$\begin{aligned} m_3 - m_2 - m_1 &= 2 \delta m_3, \quad \text{for } a = 0, \\ m_3 + 2m_2 - m_1 &= 2 \delta m_3, \quad \text{for } b = 0, \end{aligned} \quad (18)$$

where $\delta m_3 \equiv m_3 - m_3^0$ and we have used $m_2 = m_2^0$. The sum rules for $\xi = \pi/2, (3\pi/2)$ are

$$\begin{aligned} m_1 - m_3 + m_2 &= 0, \quad \text{for } a = 0, \\ m_1 - 2m_2 - m_3 &= 0, \quad \text{for } b = 0. \end{aligned} \quad (19)$$

C. Case-C

The case C can be realized by adding the A_4 breaking term δM_l to the charged lepton mass matrix M_l :

$$\delta M_l = \begin{pmatrix} g_1 v_1 & g_2 v_1 & 0 \\ g_1 \omega v_2 & g_2 v_2 & 0 \\ g_1 \omega^2 v_3 & g_3 v_3 & 0 \end{pmatrix}. \quad (20)$$

Taking $v_1 = v_2 = v_3$ and $g_1 = g_2 = g$, the matrix given by (20) becomes

$$\delta M_l = \begin{pmatrix} gv & gv & 0 \\ g\omega v & gv & 0 \\ g\omega^2 v & gv & 0 \end{pmatrix} = U_{CW} \begin{pmatrix} 0 & g & 0 \\ g & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sqrt{3}v. \quad (21)$$

Due to the addition of δM_l , the PMNS mixing matrix should be changed to $U_{12}^\dagger(\theta, \xi)U_0^{\text{TBM}}P_\beta$.

For Case C, the sum rules are given by Eq.(10).

D. Case-D

Similarly, the case D can be achieved by adding the following matrix δM_l to the charged lepton mass matrix M_l :

$$\delta M_l = \begin{pmatrix} g_1 v_1 & 0 & g_2 v_1 \\ g_1 \omega^2 v_2 & 0 & g_2 v_2 \\ g_1 \omega v_3 & 0 & g_3 v_3 \end{pmatrix}. \quad (22)$$

Taking $v_1 = v_2 = v_3$ and $g_1 = g_2 = g$, the matrix given in (22) becomes

$$\delta M_l = \begin{pmatrix} gv & 0 & gv \\ g\omega^2 v & 0 & gv \\ g\omega v & 0 & gv \end{pmatrix} = U_{CW} \begin{pmatrix} 0 & 0 & g \\ 0 & 0 & 0 \\ g & 0 & 0 \end{pmatrix} \sqrt{3}v. \quad (23)$$

The addition of δM_l causes the PMNS mixing matrix changed to $U_{13}^\dagger(\theta, \xi)U_0^{\text{TBM}}P_\beta$.

For Case D, the sum rules are given by Eq.(10).

III. PREDICTIONS OF DIRAC-TYPE CP PHASE

Now, let us review how to predict Dirac-type CP phase in PMNS mixing matrix with regards to neutrino mixing angles presented in [7]. Multiplying V given in Eq.(1) by phase matrices P_α and P_β that can be arisen from the charged lepton sector and neutrino sector, respectively, we can equate it with the standard parameterization of the PMN mixing matrix

as follows:

$$P_\alpha \cdot V \cdot P_\beta = U^{\text{ST}} = U_0^{\text{PMNS}} \cdot P_\phi$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_D} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_D} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_D} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_D} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_D} & c_{23}c_{13} \end{pmatrix} P_\phi. \quad (24)$$

The equivalence between both parameterizations dictates the following relations,

$$V_{ij}e^{i(\alpha_i+\beta_j)} = U_{ij}^{\text{ST}} = (U_0^{\text{PMNS}})_{ij}e^{i\phi_j}. \quad (25)$$

A. Case A and B

Applying $|V_{13}| = |U_{13}^{\text{ST}}|$ and $|V_{11}/V_{12}| = |U_{11}^{\text{ST}}/U_{12}^{\text{ST}}|$, we obtain the relations

$$\sin^2 \theta = 3s_{13}^2,$$

$$\cos^2 \theta = \begin{cases} 2 \tan^2 \theta_{12} & (\text{CaseA}), \\ \frac{1}{2} \cot^2 \theta_{12} & (\text{CaseB}), \end{cases} \quad (26)$$

which lead to the relation between the solar and reactor mixing angles,

$$s_{12}^2 = \begin{cases} 1 - \frac{2}{3(1-s_{13}^2)} & (\text{CaseA}), \\ \frac{1}{3(1-s_{13}^2)} & (\text{CaseB}). \end{cases} \quad (27)$$

Those relations indicate that non-zero values of s_{13}^2 lead to $s_{12}^2 < 1/3$ for Case A and $s_{12}^2 > 1/3$ for Case B. From $|V_{23}/V_{33}| = |U_{23}^{\text{ST}}/U_{33}^{\text{ST}}|$, we also get the relations

$$|\cos \eta| = \begin{cases} \frac{c_{13}^2(s_{23}^2 - c_{23}^2)}{2s_{13}\sqrt{2-6s_{13}^2}} & (\text{CaseA}), \\ \frac{c_{13}^2(c_{23}^2 - s_{23}^2)}{s_{13}\sqrt{2-3s_{13}^2}} & (\text{CaseB}). \end{cases} \quad (28)$$

Now, we demonstrate how to derive δ_D in terms of neutrino mixing angles in the standard parametrization. From the components of the neutrino mixing matrix for Case-A, we see that

$$\frac{V_{23} + V_{33}}{V_{22} + V_{32}} = \frac{V_{13}}{V_{12}}. \quad (29)$$

From the relation (25), we get the relations

$$\frac{U_{13}^{\text{ST}}}{U_{12}^{\text{ST}}} = \frac{U_{23}^{\text{ST}} + U_{33}^{\text{ST}}e^{-i(\alpha_3-\alpha_2)}}{U_{22}^{\text{ST}} + U_{32}^{\text{ST}}e^{-i(\alpha_3-\alpha_2)}}, \quad (30)$$

$$\frac{U_{3i}^{\text{ST}}}{U_{2i}^{\text{ST}}} = \frac{V_{3i}}{V_{2i}}e^{i(\alpha_3-\alpha_2)}. \quad (31)$$

Since $V_{21} = V_{31}$,

$$e^{i(\alpha_3 - \alpha_2)} = \frac{U_{31}^{\text{ST}}}{U_{21}^{\text{ST}}}. \quad (32)$$

Plugging Eq.(32) into Eq.(30), we finally obtain the relation

$$\frac{U_{13}^{\text{ST}}}{U_{12}^{\text{ST}}} = \frac{U_{23}^{\text{ST}}U_{31}^{\text{ST}} + U_{33}^{\text{ST}}U_{21}^{\text{ST}}}{U_{22}^{\text{ST}}U_{31}^{\text{ST}} + U_{32}^{\text{ST}}U_{21}^{\text{ST}}}. \quad (33)$$

Notice that the Majorana phases in Eq.(33) are cancelled. Presenting U_{ij}^{ST} explicitly in terms of the neutrino mixing angles as well as δ_D , we get the equation for δ_D as

$$\cos \delta_D = \frac{-1}{2 \tan 2\theta_{23}} \cdot \frac{1 - 5s_{13}^2}{s_{13}\sqrt{2 - 6s_{13}^2}}. \quad (34)$$

Notice that the imaginary part in Eq. (33) is automatically cancelled.

Similarly, we get the relation for Case B,

$$\cos \delta_D = \frac{1}{2 \tan 2\theta_{23}} \cdot \frac{2 - 4s_{13}^2}{s_{13}\sqrt{2 - 3s_{13}^2}}. \quad (35)$$

B. Case C and D

Applying $|V_{13}| = |U_{13}^{\text{ST}}|$ and $|V_{23}/V_{33}| = |U_{23}^{\text{ST}}/U_{33}^{\text{ST}}|$, we obtain the relations

$$\begin{aligned} \sin^2 \theta &= 2s_{13}^2, \\ \cos^2 \theta &= \begin{cases} \tan^2 \theta_{23} & (\text{Case C}), \\ \cot^2 \theta_{23} & (\text{Case D}), \end{cases} \end{aligned} \quad (36)$$

which lead to the relation between the atmospheric and reactor mixing angles,

$$s_{23}^2 = \begin{cases} 1 - \frac{1}{2(1-s_{13}^2)} & (\text{Case C}), \\ \frac{1}{2(1-s_{13}^2)} & (\text{Case D}). \end{cases} \quad (37)$$

Those relations indicate that non-zero values of s_{13}^2 lead to $s_{23}^2 < 1/2$ for Case C and $s_{23}^2 > 1/2$ for Case D. From $|V_{11}/V_{12}| = |U_{11}^{\text{ST}}/U_{12}^{\text{ST}}|$, we also get the relation

$$|\cos \eta| = \frac{3c_{13}^2 s_{12}^2 - 1}{2s_{13}\sqrt{2 - 4s_{13}^2}}. \quad (38)$$

We note that both cases lead to the same relation for $|\cos \eta|$.

Following the same procedures for obtaining Eqs.(34, 35), we get the relations

$$\cos \delta_D = \begin{cases} \frac{s_{13}^2 - (1 - 3s_{12}^2)(1 - 3s_{13}^2)}{6s_{12}c_{12}s_{13}\sqrt{1 - 2s_{13}^2}} & (\text{Case C}), \\ \frac{(1 - 3s_{12}^2)(1 - 3s_{13}^2) - s_{13}^2}{6s_{12}c_{12}s_{13}\sqrt{1 - 2s_{13}^2}} & (\text{Case D}). \end{cases} \quad (39)$$

Sustituting experimental values for neutrino mixing angles into Eqs.(34,35,39), we can estimate the values of δ_D for each cases.

IV. NUMERICAL RESULTS

For our numerical analysis, we take the current experimental data for three neutrino mixing angles as inputs, which are given at $1\sigma - 3\sigma$ C.L., as presented in Ref. [5]. This analysis is, in fact, to update the numerical results for the prediction of δ_D given in [7] by taking the new fit to the data [5]. However, as shown later, the results based on 1σ data is completely different from those in [7]. Here, we perform numerical analysis and present results only for normal hierarchical neutrino mass spectrum. It is straight-forward to get numerical results for the inverted hierarchical case, and we anticipate that the conclusion is not severely changed in the inverted hierarchical case. Using experimental results for three neutrino mixing angles, we first check if the relations Eqs.(27,37) hold and then estimate the values of δ_D in terms of neutrino mixing angles for those four cases.

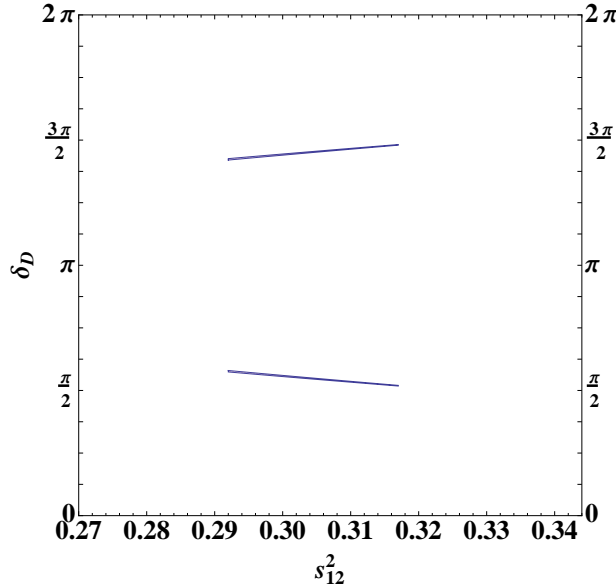


FIG. 1. Prediction of δ_D in terms of s^2_{12} for Case C based on 1σ experimental data.

A. Results for 1 σ C.L.

Plugging the experimental data for s_{13}^2 at 1σ C.L. into Eqs.(27,37), we predict the values of the mixing parameters s_{12}^2 (Case A and B) and s_{23}^2 (Case C and D) as follows:

$$s_{12}^2 = \begin{cases} 0.318 - 0.319(\text{CaseA}), \\ 0.340 - 0.341(\text{CaseB}), \end{cases} \quad (40)$$

$$s_{23}^2 = \begin{cases} 0.488 - 0.489(\text{CaseC}), \\ 0.510 - 0.511(\text{CaseD}). \end{cases} \quad (41)$$

We see that s_{12}^2 and s_{23}^2 are very narrowly determined for the 1σ region of s_{13}^2 . Comparing the experimental values of s_{12}^2 and s_{23}^2 with the above predictions, we see that only Case C is consistent with experimental results at 1σ C.L.

In Fig. 1, we show the prediction of δ_D in terms of s_{12}^2 based on the experimental data at 1σ C.L. The upper curve in Fig. 1 indicates $1.32\pi \leq \delta_D \leq 1.52\pi$ which is consistent with the result of fit for CP phase ($1.3\pi \leq \delta_D \leq 1.92\pi$) shown in [5].

B. Results for 3 σ C.L.

Plugging the experimental data for $\sin^2 \theta_{13}$ at 3σ C.L. into Eqs.(27,37), we predict the values of the mixing parameters $\sin^2 \theta_{12}$ (Case A and B) and $\sin^2 \theta_{23}$ (Case C and D) as follows:

$$s_{12}^2 = \begin{cases} 0.316 - 0.321(\text{CaseA}), \\ 0.340 - 0.342(\text{CaseB}), \end{cases} \quad (42)$$

$$s_{23}^2 = \begin{cases} 0.487 - 0.491(\text{CaseC}), \\ 0.509 - 0.513(\text{CaseD}). \end{cases} \quad (43)$$

Comparing the experimental values of s_{12}^2 and s_{23}^2 with the above predictions, we see that they are all consistent with experimental results at 3σ C.L. In particular, the prediction of s_{12}^2 for Case B prefers to nearly upper limit of 3σ allowed region.

Fig. 2 shows the predictions of δ_D in terms of s_{23}^2 ((a): Cases A and B) and s_{12}^2 ((b): Cases C and D) based on the corresponding experimental data given at 3σ C.L. Regions surrounded by blue and red lines correspond to Cases (A, C) and (B, D), respectively. The width of each bands implies the variation of the other mixing angles, s_{12}^2 (Cases A and B)

and s_{23}^2 (Cases C and D). We see that almost maximal $\delta_D \sim \pi/2, 3\pi/2$ can be achieved by $s_{23}^2 \sim 0.5$ for Cases–A, B and by $s_{12}^2 \sim 0.325$ for Cases–C,D . The values around $3\pi/2$ is consistent with the current fit of the Dirac type CP phase [5].

Comparing those results with the corresponding ones presented in [7], we see that the shapes of the curves in each cases are nearly unchanged, but the widths of each bands get much narrower. The allowed regions of s_{12}^2 above 0.344 for Cases C and D are excluded in the updated analysis.

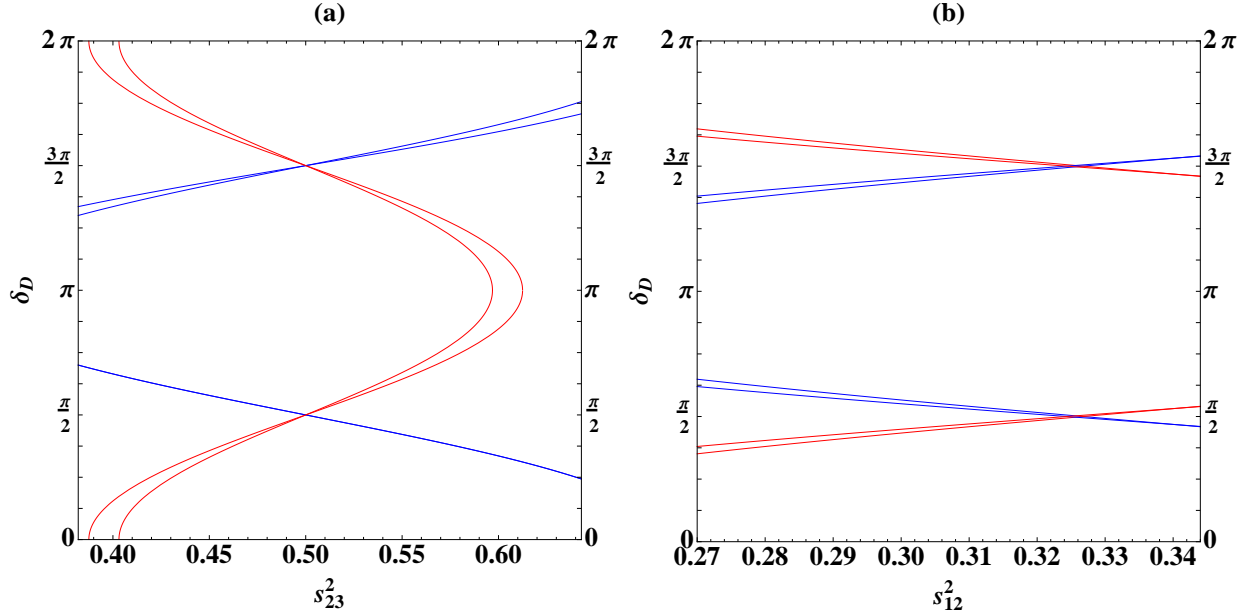


FIG. 2. Prediction of δ_D in terms of (a) s_{23}^2 for Cases A and B, and (b) s_{12}^2 for Cases C and D based on 3σ experimental data. Regions surrounded by blue (red) lines correspond to Cases A, C and (B, D).

V. CONCLUSION

As a summary, we have considered how non-zero mixing angle θ_{13} and CP violation can be accommodated in a model with A_4 flavor symmetry by incorporating symmetry breaking terms appropriately. The four possible forms of neutrino mixing matrix we considered are minimal modifications to TBM mixing matrix and factorized by TBM mixing form and an unitary mixing matrix with an angle and a CP phase corresponding to a rotation in a plane. We have shown that possible size of the Dirac-type CP phase δ_D can be predicted

with regards to two neutrino mixing angles in the standard parametrization of the neutrino mixing matrix. This has been achieved by equating one of minimally modified TBM mixing matrix with the standard parametrization of the PMNS one. Based on the current fit results for the neutrino mixing angles and CP phase, we have seen that the neutrino mixing matrix corresponding to Case C is consistent with the current fit data at 1σ C.L. whereas others are not so. This result is different from that in [7]. Extending the analysis to 3σ C.L., all cases are consistent with the current fit data. We have presented the numerical results for the predictions of δ_D in terms of either s_{12}^2 or s_{23}^2 for those cases.

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